Laser Lecture 6 Threshold Power & Four Level Laser System

The threshold pump power of a laser is the value of the pump power at which the laser threshold is just reached, assuming steady-state conditions. At this point a laser's output is dominated by stimulated emission rather than by spontaneous emission and the small-signal gain equals the losses of the laser resonator.

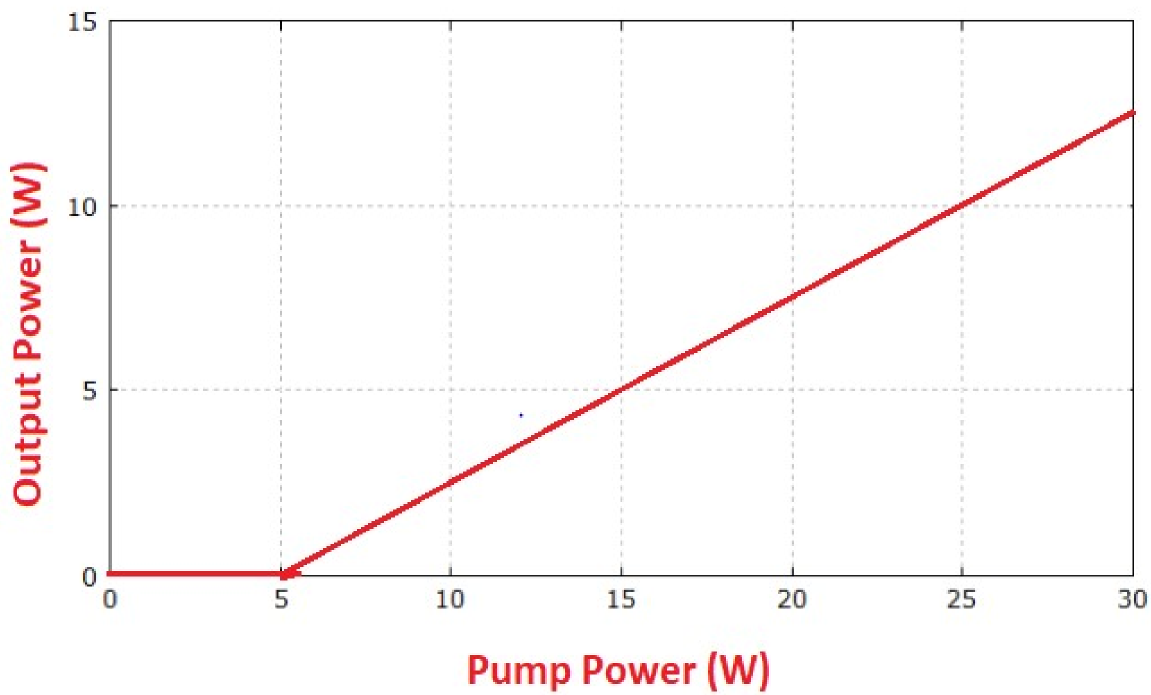
The stimulated emission cross section is given by

$$\sigma_{21}(\nu) = A_{21} \frac{\lambda^2}{8\pi n^2} g(\nu)$$

A_{21} is the Einstein coefficient
λ is the wavelength in vacuum
n is the refractive index of the medium (dimensionless), and
$g(v)$ is the spectral line shape function

A low threshold power requires low resonator losses and a high gain efficiency.

Figure : Characteristic Output versus input power for an optically pumped laser. The threshold pump power is assumed 5 W.



https://www.rp-photonics.com/threshold_pump_power.html#:~:text=The%20threshold%20pump%20power%20of%20a%20laser%20is,threshold%20is%20just%20reached%2C%20usually%20assuming%20steady-state%20conditions.

Although energy generated by stimulated emission is always at the exact frequency of the field which has stimulated it, the rate equations refers only to excitation at the particular optical frequency ν_o corresponding to the energy of the transition. At frequencies away from ν_o the strength of stimulated emission will be decreased according to the line shape function. Considering only homogeneous broadening affecting an atomic or molecular resonance, the spectral line shape function is described as a Lorentzian distribution

$$g'(\nu) = \frac{1}{\pi} \frac{(\Gamma/2)}{(\nu - \nu_0)^2 + (\Gamma/2)^2}$$

here Γ is the full width at half maximum or FWHM bandwidth

The peak value of the Lorentzian line shape occurs at the line center, $\nu = \nu_o$. A line shape function can be normalized so that its value at ν_o is unity; in the case of a Lorentzian we obtain

$$g(\nu) = \frac{g'(\nu)}{g'(\nu_0)} = \frac{(\Gamma/2)^2}{(\nu - \nu_0)^2 + (\Gamma/2)^2}.$$

Thus stimulated emission at frequencies away from ν_o is reduced by this factor. In practice there may also be broadening of the line shape due to inhomogeneous broadening, most notably due to the Doppler effect resulting from the distribution of velocities in a gas at a certain temperature. This has a Gaussian shape and reduces the peak strength of the line shape function

The Threshold Pumping Power

The threshold inversion required is usually very small compared to N (i.e. $N_2 - N_1 << N$)

The threshold value of W_p required to start laser oscillation, $W_p \approx T_{21}$

Number of atoms being pumped from 1 to 3 per unit time per unit volume is $W_p N_1$

If ν_p represents average pump frequency corresponding to excitation $E_1 \rightarrow E_3$, the power required per unit volume will be

$$P = W_p N_1 h \nu_p$$

Thus the threshold pump power for laser oscillation

$$P = T_{21} N_1 h \nu_p$$

Since N_2-N_1 << N and $N_3 \approx 0$, $N_1 \approx N_2 \approx N/2$

Also assuming $A_{21} >> S_{21}$

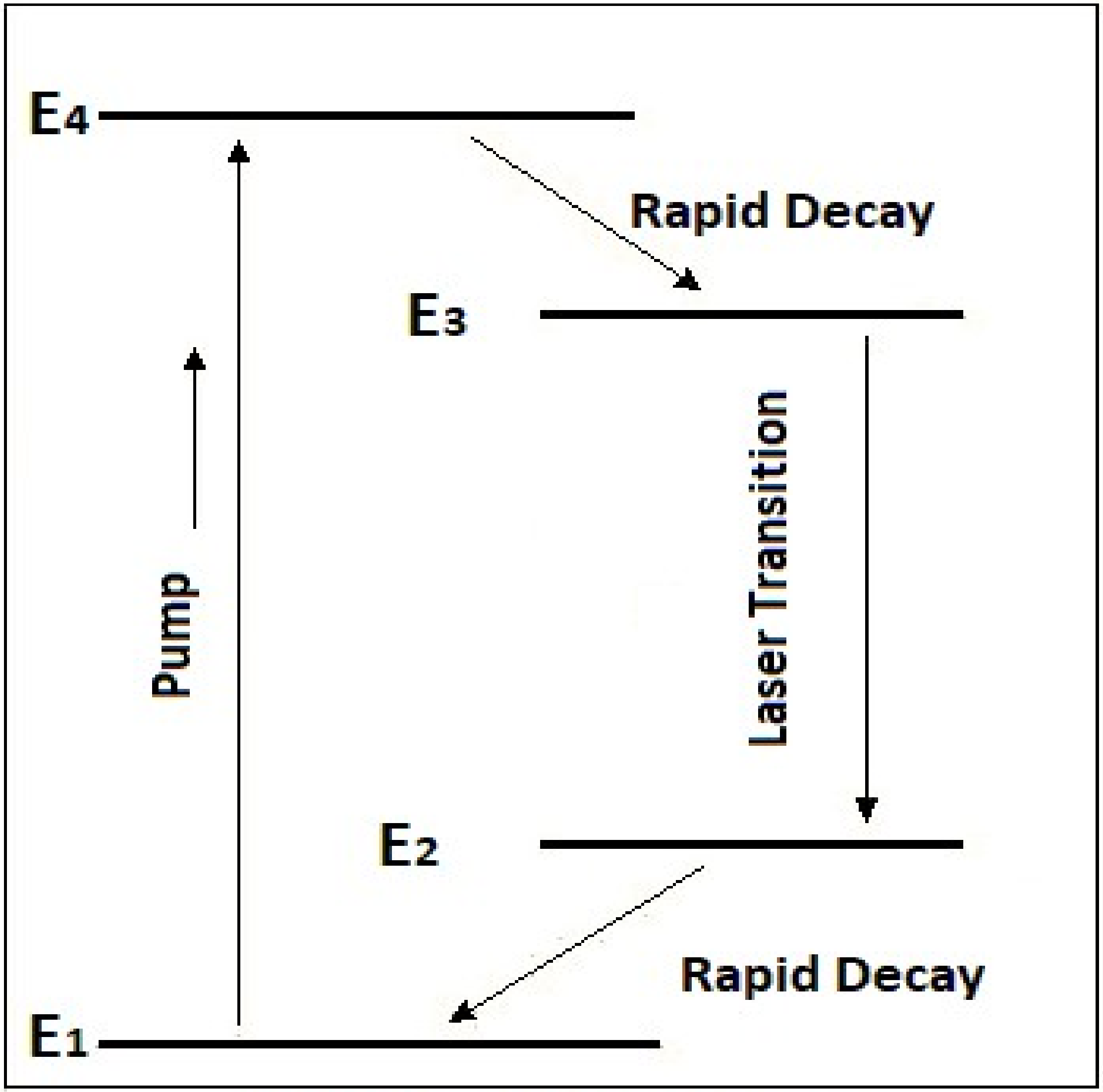
$$P \approx (N/2)\, h\, \nu_p\, (1/t_{sp})$$

For ruby laser

$N = 1.6 \times 10^{19}\ cm^{-3}$ $T_{sp} = 3 \times 10^{-3}$ sec

$\nu_p = 6.25 \times 10^{14}$ Hz $P = 1000\ Wcm^{-3}$

Four Level Laser System



Four Level Laser System

N_1, N_2, N_3 and N_4 are population densities of energy levels E_1, E_2, E_3 and E_4 respectively.

The rate equation for level 4 will be

$$\frac{dN_4}{dt} = W_p(N_1 - N_4) - T_{43}N_4 \qquad (1)$$

W_p N_1 = rate of pumping per unit volume from 1 to 4

$T_{43} = A_{32} + S_{43}$ (2)

T_{42} and T_{41} have been neglected compared to T_{43} i.e. atoms in level 4 relax to level 3 rather than to levels 2 or 1

Rate equation for level 3

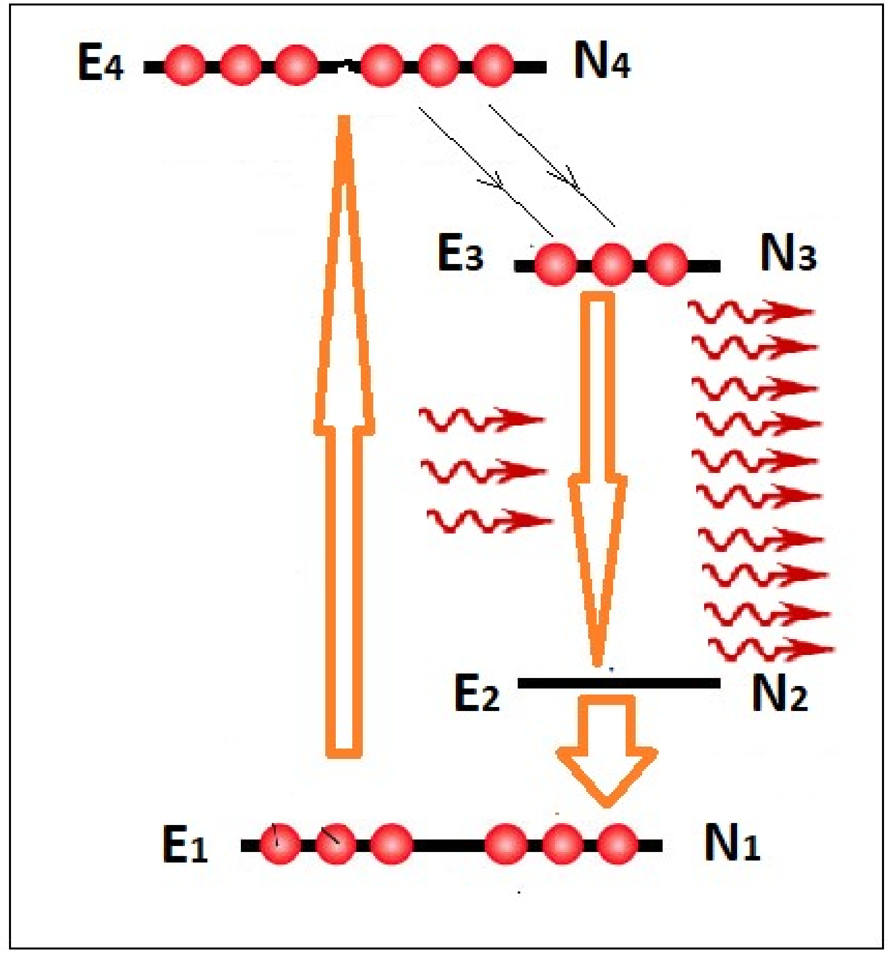
$$\frac{dN_3}{dt} = W_1(N_2 - N_3) + T_{43}N_4 - T_{32}N_3$$

(3)

Here

$$W_1 = \frac{\pi^2 c^2}{\hbar\omega^2 n_o^2} A_{32} g_1(\omega) I_1$$

(4)



W_1 = stimulated transition rate per atom between levels 3 and 2 and subscript 1 stands for laser transition

$g_1(\omega)$= line shape function fro transition 3→2

I_1 is the intensity of radiation of radiation at frequency $\omega=(E_2-E_1)/\hbar$

$T_{32} = A_{32} + S_{32}$ (5)

In equation (3) spontaneous transition from level 3 to level 1 has been neglected

$$\frac{dN_2}{dt} = -W_1(N_2 - N_3) + T_{32}N_3 - T_{21}N_2 \qquad (6)$$

$T_{21} = A_{21} + S_{21}$ (7)

Also,

Under steady state condition

$$\frac{dN_1}{dt} = 0 = \frac{dN_2}{dt} = \frac{dN_3}{dt} = \frac{dN_4}{dt} \qquad (8)$$

$$\frac{dN_4}{dt} = 0 \ \rightarrow \frac{N_4}{N_1} = \frac{W_p}{W_p + T_{43}} \qquad (9)$$

Relaxation from level 4 to 3 is very rapid. Hence

$T_{43} >> W_p \qquad N_4 << N_1 \qquad (10)$

Using approximations (10) in equations (4), (6), (8)

$$\frac{N_3 \quad N_2}{N} = \frac{W_p(T_{21} \quad T_{32})}{W_p(T_{21} + T_{32}) + T_{32}T_{21} + W_1(2W_p + T_{21})} \qquad (11)$$

In order to be able to obtain population inversion between levels 3 and 2

$T_{21} > T_{32}$ i.e spontaneous rate of de-excitation from level 2 to level 1 must be greater than the spontaneous rate of transition between level 3 to level 2

If we assume $T_{21} >> T_{32}$

$$\frac{N_3 - N_2}{N} = \frac{W_p(T_{21} - \cancel{T_{32}})}{W_p(T_{21} + \cancel{T_{32}}) + T_{32}T_{21} + W_1(2W_p + T_{21})} \qquad (12)$$

$$\frac{N_3 - N_2}{N} = \frac{W_p}{W_p + T_{32}} \frac{1}{1 + W_1(T_{21} + 2W_p)/T_{21}(W_p + T_{32})} \qquad (13)$$

From Eqn (13) it is clear that even for a small pump rate one can obtain population inversion between levels 3 and 2

The factor in red in equation (13) which is independent of W_1 gives small signal gain coefficient

The factor in green in equation (15) gives the saturation behaviour

Just below the threshold for laser oscillation, $W_1 \approx 0$

Thus

$$\frac{\Delta N}{N} = \frac{W_p}{W_p + T_{32}} \qquad (14)$$

YouTube Channel Link:
https://www.youtube.com/channel/UC3rdRYA605bdDdsJdEf0oJw/featured

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